Sampling variance in sale lots and its influence on test prediction.

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Sampling Variance in Sale Lots and its Influence on Test Precision

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Summary

Some aspects of sampling raw wool sale lots are examined. In particular, the importance of within-bale variance to the design of sampling schedules and sampling precision is considered, and the most recent estimates of this parameter are reviewed. The estimate of within-bale variance of wool-base used in IWTO and Australian core-sampling standards is likely to be exceeded by around one-third of the Australian clip, especially non-fleece types.

The importance of sampling variance to overall test precision is reviewed, and the effect of altering the sampling schedule on the precision of the combined certificate for consignments is determined. Increased sampling results in only a modest improvement in precision at both the lot and consignment level. The relevance of specifying a minimum sampling precision for sale lots is discussed, and it is argued that regular monitoring of all components of testing variance would be desirable.

Keywords: Sampling variance, test precision.

Introduction

Over 99% of wool auctioned in Australia is traded on the basis of a core test result. Woolbase (WB), vegetable matter (VM) and fibre diameter (FD) are all estimated from a core sample obtained from each lot of wool presented for sale. Core sampling involves driving a long hollow 3/4" tube through each bale and withdrawing a sample that traverses over 94% of the entire bale length. All cores from a lot are composited, with the resulting sample blended and sub-sampled prior to testing. Generally the core sampling process is completed on behalf of the woolgrower by the selling agent under the supervision of a testing organisation, eg. Australian Wool Testing Authority Ltd.

The number of cores taken and the method of withdrawal have important implications for the reliability of the test result. The IWTO Core Test Regulations and the Australian Standard for core sampling of raw wool (AS 1980-1982) describe procedures to be followed during sampling, specification limits on coring apparatus, and provides a schedule for the number of cores per bale that need to be taken in order to satisfy the precision requirements of the standard. This paper outlines the factors that affect the precision of the sampling schedule, particularly within-bale variance of wool base, and reviews previous estimates of this parameter. The adequacy of the current sampling schedule with respect to the precision requirements of the standard is investigated, and the effects of sampling variance on the precision of the test result at both lot and consignment level are also considered.
Factors Affecting Sampling Precision and Variance

The precision of a sampling schedule is a measure of the amount of variation in the mean result that would be expected to occur if the sampling process were repeated a number of times on the same lot of wool. In other words, precision describes the closeness of repeated samples to each other, and by inference, to the true mean of the lot. In order to determine the precision of a given sampling schedule one must first estimate the sampling variance for the character of interest, which is given (Cameron 1951) by:

\[
\text{Var}(\bar{y}_s) = \frac{(N-n) \sigma_b^2}{Nn} + \frac{\sigma_w^2}{nc}
\]  

Where: \(\text{Var}(\bar{y}_s)\) = sampling variance of the lot mean  
\(N\) = total number of bales in the lot  
\(n\) = number of bales in the lot from which samples are drawn  
\(\sigma_b^2\) = between bales component of variance in the characteristic \(y\)  
\(\sigma_w^2\) = within bales component of variance in the characteristic \(y\)  
\(c\) = number of cores drawn from each bale

Current core sampling standards specify that every bale in the lot must be sampled. Hence \(N = n\), reducing (1) to:

\[
\text{Var}(\bar{y}_s) = \frac{\sigma_w^2}{nc}
\]  

The precision of the sampling schedule is normally expressed as a confidence interval about the true lot mean, with a low value representing a high level of precision. The confidence interval \((\pm E)\) expected from a given sampling plan, with a probability of 0.95 (i.e. 95 times out of 100) can be predicted from (2).

\[
E = 1.96 \sqrt{\text{Var}(\bar{y}_s)}
\]

\[
E = 1.96 \sqrt{\frac{\sigma_w^2}{nc}}
\]  

where 1.96 is the normal deviate exceeded with probability 0.025 (corresponding to a 95% confidence interval). Therefore the precision level that we could expect to obtain from a given sampling plan is dependent on the number of bales in the lot, the number of cores drawn from each bale and the degree of variation that exists within bales in the lot (the within-bale variance). It will increase as the total number of cores taken from the lot (nc) increases, or as the \(\sigma_w^2\) decreases. If two lots of wool were sampled equally (i.e. the same number of cores taken), the lot with the lowest \(\sigma_w^2\) would be sampled with the greatest precision.

The core sampling schedule in both the IWTO and Australian Standards is designed to produce a sample having a precision of no worse that 1% IWTO Clean Wool Content (equal to 0.835% wool base) with a probability of 0.95. The minimum number of cores to be taken from a lot to produce the required precision can therefore be determined from (4).
Assuming a required precision level of 0.835% wool base, the number of cores to be taken from a lot is given by:

\[
nc = \left( \frac{1.96}{0.835} \right)^2 \sigma_w^2
\]

\[
= 5.51 \sigma_w^2 \quad ............. \tag{5}
\]

Thus, the within-bale variance of wool base is the main determinant of the minimum number of cores to be taken, and estimates of this parameter are required to determine the appropriate sampling schedule.

**What is the Within-Bale Variance of Australian Wools?**

Surveys providing estimates of within-bale variance for yield, FD and VM have been reported by Skinner (1965), Douglas (1965) and David (1968). Each of these studies however, was based on a small number of lots, and thus limited conclusions could be drawn as to the size and distribution of within-bale variance of Australian wools. As part of the Australian Objective Measurement Project (AOMP) in the early 1970's, preceding the introduction of pre-sale measurement, an extensive study was undertaken in an effort to estimate variance components of the major raw wool characteristics (David and Deltoer 1973).

This study involved the sampling and testing of cores from 946 lots of wool from five sale centres across Australia. Samples were taken in most wool-growing districts from wool covering a range of types considered to be representative. From each bale, eight cores were taken of which six were individually tested, and within- and between-bale variance components were estimated using analysis of variance procedures outlined by Cameron (1951).

David and Deltoer (1973), noting the non-normal and highly skewed distribution of \( \sigma_w^2 \) for both WB and FD applied a logarithmic transformation to those characteristics prior to further analysis. The transformations used were \( \text{FDLOG} = \log_{10}(1000 \times \sigma_w^2) \), and \( \text{WBLOG} = \log_{10}(10 \times \sigma_w^2) \). The transformation for WB was successful in achieving a normal distribution as measured by the W statistic of Shapiro and Wilk (1965). The transformation for FD was less successful, with the transformed data negatively skewed but closer to a normal distribution than the data on the original scale.

The estimates of \( \sigma_w^2 \) for WB, FD and VM for Merino and Crossbred fleece and skirting wools are shown in Table 1. The values reported by David and Deltoer (1973) were 'most likely' and '95% less than' figures. The 'most likely' value was determined through de-transformation of the mean of the transformed variable. The '95% less than' figure was calculated by de-transformation of \( \mu + 1.64 \sigma \), where \( \mu \) and \( \sigma \) were the mean and standard deviation of the transformed variable, and 1.64 is the normal deviate exceeded with probability 0.05.

The figures in Table 1 confirmed a number of trends that had been suggested by earlier work. Fleece wools have a lower \( \sigma_w^2 \) for WB and VM than skirting wools, while there is little difference between fleece and skirting wools for FD. Crossbred wools have lower \( \sigma_w^2 \) than Merino wools for both WB and VM, while the opposite holds for FD. Regression analysis showed a relationship existed between the mean and within-bale variance for all three characteristics examined.
Sampling variance in sale lots and its influence on test precision

Table 1 Summary of $\sigma^2$ of WB, FD and VM from all sale centres (from David and Deltoer 1973, David 1988).

<table>
<thead>
<tr>
<th>Wool Description</th>
<th>No. Lots</th>
<th>Wool Base (%)</th>
<th>Fibre Diameter (μm)</th>
<th>Veg. Matter (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Most Likely</td>
<td>95% Below</td>
<td>Most Likely</td>
</tr>
<tr>
<td>Merino Fleece</td>
<td>369</td>
<td>1.656</td>
<td>4.937</td>
<td>0.077</td>
</tr>
<tr>
<td>Crossbred Fleece</td>
<td>100</td>
<td>0.981</td>
<td>3.565</td>
<td>0.133</td>
</tr>
<tr>
<td>All Fleece</td>
<td>469</td>
<td>1.481</td>
<td>4.865</td>
<td>0.086</td>
</tr>
<tr>
<td>Merino Skirtings</td>
<td>329</td>
<td>2.762</td>
<td>9.240</td>
<td>0.076</td>
</tr>
<tr>
<td>Crossbred Skirtings</td>
<td>53</td>
<td>2.009</td>
<td>7.766</td>
<td>0.151</td>
</tr>
<tr>
<td>All Skirtings</td>
<td>382</td>
<td>2.642</td>
<td>9.135</td>
<td>0.084</td>
</tr>
<tr>
<td>Miscellaneous</td>
<td>95</td>
<td>1.921</td>
<td>8.775</td>
<td>0.068</td>
</tr>
<tr>
<td>Overall</td>
<td>946</td>
<td>1.920</td>
<td>7.213</td>
<td>0.083</td>
</tr>
</tbody>
</table>

The ‘most likely’ values in Table 1 represent the geometric mean on the original scale, not the arithmetic mean. When the transformed data are normally distributed, the geometric mean is also the mode or the value of highest frequency which is seldom used as a measure of central tendency. It is possible to calculate the expected arithmetic mean from the mean and variance of the transformed data, assuming normality (see Appendix 1). This figure is comparable to the arithmetic mean determined by averaging on the original scale, but is less susceptible to distortion by a small number of extreme values which occur in this data set. The derived arithmetic means of $\sigma^2$ of WB and FD are shown in Table 2.

Table 2 Mean within-bale variance of wool base and fibre diameter (re-analysis of data from survey of David and Deltoer 1973)

<table>
<thead>
<tr>
<th>Wool Description</th>
<th>Wool Base (%)</th>
<th>Fibre Diameter (μm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Merino Fleece</td>
<td>2.071</td>
<td>0.097</td>
</tr>
<tr>
<td>Merino Skirtings</td>
<td>3.591</td>
<td>0.095</td>
</tr>
<tr>
<td>Crossbred Fleece</td>
<td>1.337</td>
<td>0.176</td>
</tr>
<tr>
<td>Crossbred Skirtings</td>
<td>2.816</td>
<td>0.182</td>
</tr>
<tr>
<td>Miscellaneous</td>
<td>2.949</td>
<td>0.096</td>
</tr>
<tr>
<td>Overall</td>
<td>2.654</td>
<td>0.109</td>
</tr>
</tbody>
</table>

The arithmetic means are considerably higher than the ‘most likely’ values shown in Table 1. The pattern of differences between Merino and Crossbred wool and fleece and skirtings are similar.

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Within-Bale Variance Estimates and Sampling Precision

The number of cores to be drawn from a sale lot is determined by three requirements:
(i) The sample has a precision of no less than 1% IWTO Clean Wool Content with a probability of 0.95,
(ii) The total core mass is ≥ 750 g, and
(iii) All bales are sampled and have equal numbers of cores taken.

It is simple to design a sampling schedule that satisfies requirements (ii) and (iii), however the need to attain a given precision level presents problems. The number of cores to be drawn from a lot to obtain a sample with the required precision is determined solely by the estimate of $\sigma_w^2$ of WB (see equation 5). Core sampling standards specify a value of 2.76 for $\sigma_w^2$, this being the highest 'most likely' value reported by David and Deltour (1973), and was for Merino skirtings (see Table 1). Using this value in the standard results in a minimum number of cores to satisfy the precision requirements of 16. Depending on the average core mass and the number of bales in the lot, the actual number of cores taken, in practice may be more. However, for lot sizes of up to 10 bales, and an average core mass of 40g, 20 cores per lot will satisfy each of the three conditions required for a core sample.

If the $\sigma_w^2$ of WB in a lot is greater than 2.76, the precision level achieved by the sampling schedule may be less than that required by the standard. Table 3 shows a number of commonly occurring wool types (not necessarily a sample representative of the Australian clip) that were sampled during the AOMP, and the (arithmetic) average $\sigma_w^2$ of WB for those types. Also shown in Table 3 is the precision level to be expected from a sampling schedule drawing 16 and 20 cores per lot respectively.

Table 3 Average within-bale variance of wool base of some common wool types, and the precision level ($\pm E$ with probability 0.95) expected from sampling 16 or 20 cores per lot.

<table>
<thead>
<tr>
<th>AWC Type/s</th>
<th>Description</th>
<th>$\sigma_w^2$</th>
<th>16</th>
<th>20</th>
</tr>
</thead>
<tbody>
<tr>
<td>37/44/50</td>
<td>Merino Fleece (Spinners)</td>
<td>2.31</td>
<td>0.74</td>
<td>0.67</td>
</tr>
<tr>
<td>57/63/68</td>
<td>Merino Fleece (Best T/M)</td>
<td>1.19</td>
<td>0.53</td>
<td>0.48</td>
</tr>
<tr>
<td>73/79/85</td>
<td>Merino Fleece (Good T/M)</td>
<td>2.62</td>
<td>0.79</td>
<td>0.71</td>
</tr>
<tr>
<td>163/163A</td>
<td>Merino Skirtings (Ave. Short Lnth)</td>
<td>3.82</td>
<td>0.96</td>
<td>0.86</td>
</tr>
<tr>
<td>167</td>
<td>Merino Skirtings (Short Lnth)</td>
<td>3.96</td>
<td>0.98</td>
<td>0.87</td>
</tr>
<tr>
<td>Various</td>
<td>Merino Crutchings</td>
<td>5.59</td>
<td>1.16</td>
<td>1.04</td>
</tr>
<tr>
<td>Various</td>
<td>Merino Locks</td>
<td>3.38</td>
<td>0.90</td>
<td>0.81</td>
</tr>
<tr>
<td>433/434</td>
<td>XB Fleece (Good Style, Lnth)</td>
<td>1.68</td>
<td>0.64</td>
<td>0.57</td>
</tr>
<tr>
<td>486/487/493</td>
<td>XB Skirtings (Ave./Short Lnth)</td>
<td>3.34</td>
<td>0.90</td>
<td>0.80</td>
</tr>
</tbody>
</table>

Table 3 illustrates that a number of common wool types have an average $\sigma_w^2$ of WB greater than 2.76 and consequently, the sampling precision of these lots may be less (depending on the number of cores taken) than the required level in the standard of 0.835% wool base. The non-fleece lines all have estimates substantially greater than 2.76.
Taking 20 cores per lot in a number of cases attains the required precision level while taking 16 cores does not. For example, Merino locks and average/short length Crossbred skirtings will on average produce a sample outside the required precision level if 16 cores per lot are taken. If 20 cores are taken from the lot, the sample falls (narrowly) within the level required by the standard.

There is a certain risk in categorising a population by its mean value, especially when the distribution is highly skewed, as in this case. As with any population, a proportion of lots will be above and below the average value. Here, the population is positively skewed, and there are more lots with a within-bale variance below the average than above it.

The proportion of Australian wools which have a \( \sigma^2_w \) of WB higher than the value of 2.76 specified in the core sampling standard can be calculated from the data of David and Deltoer (1973). The log transformation of wool base is normally distributed (or close to it), so the probability of a lot randomly sampled from the population having a \( \sigma^2_w \) of WB greater than 2.76 can be determined using the standard normal variate, Z. The formula for Z is given by:

\[
Z = \frac{X - \mu}{\sigma} \quad \ldots \quad (6)
\]

Now \( X = 1.44 = \log_{10}(10 \times 2.76) \)
\( \mu = 1.28 \) = mean of \( \log_{10} \) transformed variable
\( \sigma = 0.33 \) = between lot standard deviation of \( \log_{10} \) transformed variable

\[
\therefore \quad Z = \frac{1.44 - 1.28}{0.33} = 0.47
\]

\( P(Z > 0.47) = 0.32 \)

Assuming the results of David and Deltoer (1973) are representative of the Australian clip, we can say that the probability that a randomly selected lot will have a \( \sigma^2_w \) of WB of greater than 2.76 is 0.32, i.e. approximately one-third of the clip has a \( \sigma^2_w \) greater than the value specified in both the Australian and IWTO core sampling schedules.

**The Relative Importance of Sampling Variance to Test Precision**

The minimum precision level specified in the core sampling standards refers to sampling precision only. The overall precision of the test result is also affected by variation between and within testing laboratories. Assuming there is no variation due to inadequate blending, the variance of the mean test result for WB is approximated by

\[
Var(\bar{Y}) = \frac{\sigma^2_w}{nc} + \sigma^2_L + \frac{\sigma^2_s}{s} \quad \ldots \quad (7) \quad (from \ David \ and \ Brown \ 1975)
\]

Where

\( \sigma^2_L \) = between laboratories component of variance

\( \sigma^2_s \) = within laboratories (between sub-samples) component of variance

\( s \) = number of sub-samples tested from the blended core sample
The magnitude of the three variance components above will determine the relative importance of sampling variance to the overall test precision. A number of studies have been conducted to determine this, producing variable results. David (1972, 1973a) using interlaboratory round trial data, found the within-laboratories variance to be greater than the variance between laboratories, while David and Brown (1975) using a larger set of routine test data found the opposite. All three studies found that, as with sampling variance, the between and within laboratories variance components are heterogeneous, varying according to mean WB.

The results of David and Brown (1975), divided into WB classes are shown in Table 4. Also shown is the expected precision of a WB test when twenty, forty and sixty cores per lot are taken, assuming two sub-samples are tested.

Table 4 Estimates of Variance Components for Mean WB and the Expected Precision (95% Confidence Interval) of the Test when 20, 40 and 60 Cores per Lot are Taken.

<table>
<thead>
<tr>
<th>WB class</th>
<th>$\sigma_w^2$</th>
<th>$\sigma_l^2$</th>
<th>$\sigma_s^2$</th>
<th>nc = 20</th>
<th>nc = 40</th>
<th>nc = 60</th>
</tr>
</thead>
<tbody>
<tr>
<td>&lt;45%</td>
<td>4.124</td>
<td>0.861</td>
<td>0.386</td>
<td>2.20</td>
<td>2.11</td>
<td>2.08</td>
</tr>
<tr>
<td>45-55%</td>
<td>3.078</td>
<td>0.340</td>
<td>0.163</td>
<td>1.49</td>
<td>1.38</td>
<td>1.35</td>
</tr>
<tr>
<td>55%&gt;</td>
<td>1.707</td>
<td>0.079</td>
<td>0.084</td>
<td>0.89</td>
<td>0.79</td>
<td>0.76</td>
</tr>
</tbody>
</table>

The dominant factor in determining the precision of the test (defined in eqn. 7) is the variance between laboratories, especially in low yielding wools. The gain in precision (% of WB) from taking 40 cores instead of 20 ranged from 4% in wools with low WB to 11% in wools with a higher WB, while taking 60 cores per lot produced only a slight marginal improvement. Increasing the number of sub-samples tested from the lot results in smaller improvements in precision than increased sampling (David and Brown 1975).

Obviously reducing between laboratory variance would be the most effective way of increasing test precision in low yielding wools. Updated estimates of all of the above variance components are needed considering the inconsistent results of these earlier studies, and the changes to procedures and test methods that have occurred since the above work was reported.

**Sampling Variance and the Precision of Combined Certificates**

As lots of wool are generally purchased as part of a larger consignment, it would be of interest to know the effect of the level of sampling of lots on the precision of the combined certificate at the consignment level. David (1973b) showed that, as the number of component lots increased, and the $\sigma_w^2$ of WB decreased, the precision of the combined certificate improved. However the effect of increased sampling of lots on the consignment precision has not been examined for a 'typical' processing batch.
The procedure for combining pre-sale test results for WB into a combined result for a consignment is given by Australian Standard AS 1362-1985 as,

$$
\bar{B}_i = \frac{\sum_{i=1}^{n} M_i B_i}{M}
$$

$$
- \sum_{i=1}^{n} k_i B_i \quad \text{where} \quad k_i = \frac{M_i}{M} \quad \ldots \quad (8)
$$

Where $\bar{B}_i$ = average WB of the consignment, $M_i$ = greasy wool weight in the $i^{th}$ lot, $M$ = greasy wool weight in all lots and $B_i$ = mean WB in the $i^{th}$ lot. The variance of a combined certificate can thus be determined.

$$
\text{Var} (\bar{B}_i) = \text{Var} \sum_{i=1}^{n} k_i B_i
$$

Assuming the means in each lot are independent and uncorrelated,

$$
\text{Var} (\bar{B}_i) = \sum_{i=1}^{n} \text{Var}(k_i B_i)
$$

$$
= \sum_{i=1}^{n} k_i^2 \text{Var}(B_i) \quad \ldots \quad (9)
$$

Now, $\text{Var}(B_i) = \frac{\sigma_w^2}{nc} + \sigma_B^2 + \frac{\sigma_k^2}{r}$

Hence, $\text{Var}(\bar{B}_i) = \sum_{i=1}^{n} k_i^2 \left( \frac{\sigma_w^2}{nc} + \sigma_B^2 + \frac{\sigma_k^2}{r} \right) \quad \ldots \quad (10)$

The effect of increasing the number of cores taken in each lot by a factor ($f$), on the variance of the consignment mean can be determined:

$$
\Delta \text{Var}(\bar{B}_i) = \sum_{i=1}^{n} k_i^2 \frac{\sigma_w^2}{nc} - \sum_{i=1}^{n} k_i^2 \frac{\sigma_w^2}{fnc}
$$

$$
= \sum_{i=1}^{n} k_i^2 \left( \frac{f-1}{fnc} \right) + \frac{\sigma_k^2}{r} \quad \ldots \quad (11)
$$

An example below illustrates the use of (11) in determining the effects of increasing the number of cores taken from a lot on the precision of a consignment mean. The types composing a typical consignment for a Chinese processor (100% fleece wool), and the values of $k$ for each lot are shown in Table 5 (Lu En Long 1993).
Table 5  A Typical Consignment for a Chinese Processor (100% Fleece Wool)

<table>
<thead>
<tr>
<th>Type</th>
<th>Bales</th>
<th>Cores</th>
<th>Mean WB (%)</th>
<th>Mass (kg)</th>
<th>$k_i$</th>
<th>$\sigma^2_{\text{WB}}$</th>
<th>$\text{Var}(B_i)^*$</th>
</tr>
</thead>
<tbody>
<tr>
<td>71B</td>
<td>12</td>
<td>24</td>
<td>56.2</td>
<td>2020</td>
<td>0.111</td>
<td>1.539</td>
<td>0.185</td>
</tr>
<tr>
<td>77</td>
<td>14</td>
<td>28</td>
<td>57.6</td>
<td>2512</td>
<td>0.138</td>
<td>1.411</td>
<td>0.171</td>
</tr>
<tr>
<td>78</td>
<td>7</td>
<td>21</td>
<td>57.4</td>
<td>1201</td>
<td>0.066</td>
<td>1.428</td>
<td>0.189</td>
</tr>
<tr>
<td>84B</td>
<td>4</td>
<td>16</td>
<td>55.1</td>
<td>619</td>
<td>0.034</td>
<td>1.649</td>
<td>0.224</td>
</tr>
<tr>
<td>78</td>
<td>4</td>
<td>16</td>
<td>57.4</td>
<td>783</td>
<td>0.043</td>
<td>1.411</td>
<td>0.209</td>
</tr>
<tr>
<td>78</td>
<td>6</td>
<td>18</td>
<td>57.4</td>
<td>1092</td>
<td>0.060</td>
<td>1.411</td>
<td>0.199</td>
</tr>
<tr>
<td>84</td>
<td>7</td>
<td>21</td>
<td>56.5</td>
<td>1274</td>
<td>0.070</td>
<td>1.511</td>
<td>0.193</td>
</tr>
<tr>
<td>62</td>
<td>11</td>
<td>22</td>
<td>61.3</td>
<td>2002</td>
<td>0.110</td>
<td>1.119</td>
<td>0.172</td>
</tr>
<tr>
<td>84</td>
<td>12</td>
<td>24</td>
<td>56.5</td>
<td>2075</td>
<td>0.114</td>
<td>1.511</td>
<td>0.184</td>
</tr>
<tr>
<td>84</td>
<td>5</td>
<td>20</td>
<td>56.5</td>
<td>855</td>
<td>0.047</td>
<td>1.511</td>
<td>0.197</td>
</tr>
<tr>
<td>78</td>
<td>9</td>
<td>18</td>
<td>57.4</td>
<td>1511</td>
<td>0.083</td>
<td>1.428</td>
<td>0.200</td>
</tr>
<tr>
<td>84</td>
<td>4</td>
<td>16</td>
<td>56.5</td>
<td>673</td>
<td>0.037</td>
<td>1.511</td>
<td>0.215</td>
</tr>
<tr>
<td>85B</td>
<td>9</td>
<td>18</td>
<td>55.2</td>
<td>1566</td>
<td>0.086</td>
<td>1.638</td>
<td>0.212</td>
</tr>
</tbody>
</table>

* testing 2 sub-samples.

Also shown in Table 5 for each lot is the number of cores that would be taken with the current sampling schedule, the expected mean WB (AWC 1993), $\sigma^2_{\text{WB}}$ of WB (estimated from regressions on mean WB) and the expected variance of the mean test result, calculated from (7).

The variance of the consignment mean obtained from sampling as per the current schedules is given by:

$$
\text{Var} (\bar{B}_i) = \sum_{i=1}^{n} k_i^2 \text{Var}(B_i) = 0.0168
$$

resulting in a 95% confidence interval for the consignment mean of ±0.254%. The reduction in the variance of the consignment mean resulting from doubling the number of cores taken from each lot ($f=2$) is given by:

$$
\Delta \text{Var} (\bar{B}_i) = \sum_{i=1}^{n} k_i^2 \frac{\sigma^2_{\text{WB}}}{2nc} = 0.0032
$$

The variance of the consignment mean when the sampling schedule is doubled is
0.0136, resulting in a 95% confidence interval of ±0.229%, a 10% improvement. For a typical blended consignment composed of 20 lots of both fleece and non-fleece wools, taking twice the number of cores from each lot results in a 7% improvement in precision of the consignment mean.

Discussion

Knowledge of the magnitude of within-bale variability is essential in the construction and evaluation of sampling schedules (Tanner and Lerner 1951). The significance of $\sigma_w^2$ of WB is due to the requirement of the IWTO and Australian core sampling standards that a certain level of sampling precision be attained for WB. Current schedules are based on the estimates made by David and Deltoer (1973). As this parameter is unlikely to be static over time, it needs regular updating. This would also evaluate whether changes in management and clip preparation procedures have produced any effect.

There are a number of reasons why this has not been done. It is difficult to estimate variance components in sale lots during routine pre-sale sampling, due to the need to keep individual cores separate for testing in order to determine within bale variance. Repetition of a similar study to David and Deltoer (1973) would now cost well in excess of $A250,000. In addition, it could be argued that within-bale variance is likely to vary substantially between years, depending on climatic conditions, and therefore the results for any one year will not necessarily be applicable for the next. This argument however, is problematic as no data exists between years to confirm or deny it.

Given the importance of the parameter, regular updated estimates of within bale variance of the major raw wool characteristics and a range of additional characteristics such as colour and resistance to compression should be of interest to the industry, and would be of particular use in any review of core sampling standards. A research project, funded by the Wool Research and Development Corporation is currently being undertaken in the Department of Wool and Animal Science at the University of New South Wales, investigating the most efficient way to do this.

A number of aspects of core sampling schedules may warrant review. The Australian and IWTO standards are quite specific in requiring the sampling precision of the core sample to be ±1% Clean Wool Content (or better). The value for $\sigma_w^2$ of WB that is used in the standard will determine the proportion of lots producing a sample that complies with the precision requirements. David and Deltoer (1973) noted that the choice of which value to use depended on whether an 'optimistic or cautious' sampling schedule was required. The decision to use the geometric mean of Merino skirtings from the AOMP study (2.76) in the standard indicates that an 'optimistic' schedule was decided upon, especially in view of the disparity between this value and the arithmetic mean for skirtings of 3.59, shown in Table 2. This may have resulted from non-technical commercial concerns regarding high levels of coring. The AOMP data suggest that almost one-third of lots have a $\sigma_w^2$ of WB of greater than 2.76, and a substantial number of these, especially non-fleece Merino types are likely to produce a sample having a precision lower than that required by the standard.

This problem could be rectified in two ways. A more rigorous sampling schedule could be designed to ensure compliance with the precision requirement by a larger proportion of lots. This could be achieved through the use of a higher value of $\sigma_w^2$ of WB such as the '95% less than' figure of 7.21, suggested by David and Deltoer (1973) which would result in a minimum of 40 cores per lot being sampled. Obviously, in many less variable lots such as fleece lines, especially those with small numbers of bales, this would result in needless over-sampling and additional bale damage. An alternative to this would be to
design a varying sampling schedule, whereby more variable wools such as skirtings have a greater number of cores taken. Again, it is unlikely that such a scheme would be accepted by the industry.

A second solution would be to relax or re-define the precision requirement of the sampling schedule. From the perspective of the processor the precision of the test result at the lot level, especially for small lots, is less important than the combined precision of the consignment. The precision requirement in the standard may be better aimed at a typical consignment rather than at individual lots. It would be possible to design a sampling schedule to achieve (on average) a minimum precision for processing batches. This would result in smaller (and less valuable) lots being sampled less rigorously than lots that make up a greater proportion of the final consignment. Also, more variable types which are usually lower in value would have a lower absolute precision but a similar precision in financial terms. However, this approach to the design of sampling schedules could result in problems over the issue of grower equity in wool sales, as well as reduced trade confidence in test results, a factor important to the recovery of the industry in a competitive marketplace.

The problem of designing a suitable sampling schedule needs to be viewed in context of the overall precision of the test result. Available estimates of variance components of the mean value for a WB test suggest that increasing the number of cores effects only a modest improvement in precision at both the lot and consignment levels. Other studies (Ward 1981; Anon 1988) have shown the improvement in FD precision resulting from increased sampling to be even smaller than that for WB. It appears that the greatest potential for increasing test precision lies with reducing variation between- and within-laboratories rather than at the sampling level. Test precision could be improved through additional sampling and testing, a process that would increase costs considerably more than increasing sampling alone.

This conclusion however should be qualified with the knowledge that there is disagreement in the literature as to the relative magnitude of the different variance components. Updating and monitoring the relevant parameters would prove informative in the evaluation of all sampling and testing procedures.

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References


Sampling variance in sale lots and its influence on test precision


Appendix 1

Calculation of arithmetic mean from log_{10} transformed data.
Assume $X \sim N(\mu, \sigma^2)$, and that $\mu$ and $\sigma^2$ are estimated by $E(X)$ and $Var(X)$ respectively.

We know that $X = \log_{10}(10 \times Y)$ (Y = $\sigma_w^2$ of WB)

$$E(X) = 0.4343 \ln Y + 1$$

We wish to calculate $E(Y)$ from $X$. If $X' = \ln Y$, then $E(Y) = e^{\mu+1/2\sigma'^2} .... (A1)$. Our equation is $X = 0.4343 \ln Y + 1$, so we must adjust the mean and variance accordingly before we can calculate the expectation of $Y$.

$X = 0.4343 X' + 1$

$\therefore X' = 2.302 (X-1)$

$E(X') + 2.302 (E(X) -1) = 2.302 (m-1)$

$Var (X') = Var (2.302 X) = (2.302)^2 Var (X) = (2.302)^2 \sigma^2$

Substitution of the values of $E(X')$ and $Var(X')$ for $\mu$ and $\sigma^2$ in Eqn A1 above will give an estimate of the expected value (average) of $Y$, i.e. $\sigma_w^2$ of WB.

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